UNIVERSIDAD POLITÉCNICA DE MADRID
POLITÉCNICA

INDUSTRIALES
ETSII | UPM

UNIVERSIDAD POLITÉCNICA DE MADRID
ESCUELA TÉCNICA SUPERIOR DE
INGENIEROS INDUSTRIALES

Differential Equations

Program

Unit 1. Solutions to Ordinary Differential Equations. Elementary methods .

1.1. Ordinary Differential Equations (ODEs). Definitions. Solutions. Initial value problems (or Cauchy problems).

1.2. Exact differential equations. Finding the potential function.

1.3. Separable equations. Homogeneous equations.

1.4. First order linear differential equations. Bernoulli equations.

1.5. Change of variable. Order reduction.

Unit 2. First order linear differential systems with constant coefficients.

2.1. First order linear differential systems with constant coefficients. Matrix notation X'=AX.

2.2. Diagonalizable matrix A on **R** or **C**. General solution to the differential system in terms of the eigenvalues and eigenvectors of the matrix A.

2.3. General case: exponential of a matrix. Calculation methods. Solution to an initial value problem.

2.4. Nonhomogeneous differential systems. Variation of constants formula.

2.5. Phase portrait of linear plane differential systems. Classification: node, saddle points, spirals, and centers.

2.6. Some examples of linear differential systems with variable coefficients.

Unit 3. Higher order linear differential equations with constant coefficients.

3.1. Higher order linear ODEs with constant coefficients. Fundamental system of solutions.

3.2. Nonhomogeneous case: variation of constants and undetermined coefficients methods.

3.3. Higher order linear ODEs with variable coefficients: order reduction. Euler equations.

Unit 4. Nonlinear differential systems.

4.1. Autonomous nonlinear differential systems. Existence and uniqueness of solution to the initial value problem. Continuation of solutions.

4.2. Orbits. Equilibria. Phase portrait and enlarged phase portrait.

4.3. First integrals.

4.4. Stable, asymptotically stable and unstable equilibria. Stability of equilibria by the linearization method. Hyperbolic equilibria. Hartman-Grossman Theorem.

4.5. Stability by the Lyapunov direct method. Lyapunov functions.

4.6. Closed orbits and limit cycles.

4.7. Plane differential systems. Poincaré, Bendixson, and Poincaré-Bendixson Theorems.

4.8. Applications: conservative mechanical systems. Conservation of energy Theorem.

4.9. Models in Ecology: Lotka-Volterra predator-prey model. Species in competition. Other examples: models in Electricity, Economics, etc.

4.10. Notions on dynamical systems depending on a parameter.

Unit 5. Introduction to the Partial Differential Equations. Separation of variables method.

5.1. Trigonometric Fourier series expansion of a periodic function.

5.2. Second order linear partial differential equations (PDEs) of two independent variables. Equations in Mathematical Physics: wave equation, Laplace equation, and heat equation.

5.3. Solution to PDEs by the separation of variables method. Eigenvalues and eigenvectors problems. Formal solution.

5.4. Boundary value plane problems in polar coordinates. Dirichlet problem for the Laplace equation. Poisson formula.

Bibliography

1. E. Sánchez, J. González, and J. Gutiérrez, *Sistemas Dinámicos. Una introducción a través de ejercicios.*Sección de Publicaciones de la ETSI Industriales de la UPM.

2. D. G. Zill and M. R. Cullen (2002), *Ecuaciones Diferenciales con Problemas de Valores en la Frontera,* Thomson Learning.

3. R. Nagle and E. Saff (1992), *Fundamentos de Ecuaciones Diferenciales,* Addison Wesley Iberoamericana.