UNIVERSIDAD POLITÉCNICA MADRID
POLITÉCNICA

INDUSTRIALES
ETSII | UPM

UNIVERSIDAD POLITÉCNICA DE MADRID
ESCUELA TÉCNICA SUPERIOR DE
INGENIEROS INDUSTRIALES

Calculus II

Program

Unit 1. Improper Integrals.

1.1. Unbounded domains of integration: definition of convergence. Non-negative integrands. Convergence criteria: comparison and asymptotic comparison tests. Integrands not necessarily positive. Cauchy test and absolutely convergent integrals.

1.2. Unbounded integrands. Definition of convergence and corresponding properties.

Unit 2. Numerical Series.

2.1. Definition and convergence. Geometric, telescopic, and harmonic series. The necessary condition of convergence. Cauchy test of convergence. Operations on series.

2.2. Series with non-negative terms. Comparison and asymptotic comparison tests. Ratio, root, and integral tests.

2.3. Alternating series. The Leibniz test and estimation of the error. The Euler constant.

2.4. Series with arbitrary terms. Absolutely convergent series. Ratio and root tests for series with arbitrary terms.

Unit 3. Power Series.

3.1. Definition. Radius of convergence. The Cauchy-Hadamard formula. Algebra of power series.

3.2. Differentiation and integration of power series. Abel’s theorem. Taylor series.

Unit 4. The Space R^n.

4.1. The vector linear space R^n. Inner product and Euclidean norm. Schwarz and Minkowski inequalities. Normed spaces. Equivalent norms. Sequences of vectors. Completeness of R^n.

4.2. Basic topology of R^n. Interior points. Open sets and their properties. Boundary points. Adherent points. Closed sets and their properties. Accumulation points. The Bolzano-Weierstrass theorem.

Unit 5. Continuity and Limits.

5.1. Limit of a function. Continuous functions. Algebra of limits and continuous functions. Limits and continuity. Continuity and convergence of sequences.

5.2. Properties of continuous functions. Compact sets, characterization. Continuous image of a compact set.

Unit 6. Differentiation.

6.1. First partial derivatives. Geometric and physical interpretation. Directional derivative along a vector. Partial derivatives of a vector valued function. Gradient.

6.2. Differentiation. Geometric and physical interpretation. Algebra of differentiable functions. Necessary conditions of differentiability. Jacobian matrix. Maximum directional derivative. Sufficient condition of differentiability.

Unit 7. The Chain Rule and Applications.

7.1. The chain rule. Jacobian matrix of composite functions. Mean value theorem. Differentiation under the integral sign: Leibniz' rule.

7.2. Scalar and vector fields in R^3. Curves in R^3. Derivative of a field along a curve. Tangent plane to a surface given by explicit and parametric equations. Tangent plane to a level surface.

Unit 8. Successive Derivatives. Taylor's Formula.

8.1. Second partial derivatives. Wave, heat, and Laplace equations. Schwarz theorem. Higher-order partial derivatives.

8.2. Hessian matrix. Taylor expansion of order two with Peano's remainder. Arbitrary Taylor expansions.

Unit 9. Implicit and Inverse Function Theorems.

9.1. Inverse function theorem: the case n=1. Inverse function of an affine map. Inverse function theorem: general case. Jacobian matrix and determinant of inverse functions.

9.2. Implicit functions. Implicit function theorem. Jacobian matrix and derivatives of implicit functions.

Unit 10. Maxima and Minima.

10.1. Extremal points. Necessary condition of first order: Fermat's theorem. Stationary points. Necessary condition of second order for a point to be extremal.

10.2. Real quadratic forms. Positive and negative definite quadratic forms. Sylvester's criterion. Sufficient condition of second order for a point to be extremal.

Unit 11. Constrained Extrema.

11.1. Problem statement. The case n=2. The Lagrange multipliers theorem.

11.2. Sufficient conditions. Application to parameter sensitivity analysis.

Bibliography

1. R. Riaza and M. Álvarez, *Cálculo Infinitesimal*, Sociedad de Amigos de la ETSI Industriales de la UPM.
2. R. Courant and F. John, *Introducción al Cálculo y al Análisis Matemático*, Ed. Limusa.
3. J. E. Marsden and A. J. Tromba, *Cálculo Vectorial*, Ed. Addison Wesley Iberoamericana.

Problem books:

1. J. Ruiz, *Cuestiones de Cálculo*, Biblioteca de la ETSI Industriales de la UPM.

2. Liashkó et al., *Matemática Superior. Problemas resueltos*, Ed. URSS.

3. P. García, R. Riaza, A. Rincón, and M. Tablada, *Problemas de Cálculo Infinitesimal. Cálculo II*, Sección de Publicaciones de ETSI Industriales de la UPM.

4. L. Fernández, P. García, A. Rincón, and M. Tablada, *Problemas de examen. Cálculo II*, Sección de Publicaciones de ETSI Industriales de la UPM.