CALCULUS I CONTENTS.

Chapter 1. The real-number system.

The natural, integer, rational and real numbers. Absolute value. Ordered sets. Inequalities and intervals. Supremum of a set of real numbers. Bounded sets. The supremum axiom.

Chapter 2. Sequences.

Sequences of real numbers. Convergence. Monotone sequences. Euler's number. Cauchy sequences. Completeness of the real-number system. Recursive sequences.

Chapter 3. Real functions.

Informal description and examples. Elementary functions. Composite functions. Continuity and limit of a function. Sequences and continuity. Zeros of functions. Bolzano's theorem. The intermediate-value theorem for continuous functions. The extreme-value theorem. Inverse of monotonic functions.

Chapter 4. Differential Calculus.

The derivative of a function. Tangent line. Geometric interpretation of the derivative as a slope. The relationship between differentiation and continuity. The algebra of derivatives. The chain rule for differentiating composite functions. Leibniz's rule. Derivatives of inverse functions. Derivatives of the elementary functions.

Chapter 5. Mean-value theorems.

Rolle's theorem. Lagrange's mean-value theorem. Applications of differentiation to extreme values of functions.

Chapter 6. Indeterminate forms. Convex functions.

L'Hôpital's rule. The indeterminate forms $0/0$ and ∞/∞. Convex functions: definition and main properties.

Chapter 7. Polynomial approximations to functions.

The Taylor polynomials generated by a function. Algebra of Taylor's polynomials. Taylor's formula with remainder. Forms for the remainder in Taylor's formula. Taylor's polynomials of elementary functions. Estimates for the error in Taylor's formula. The o-notation.

Chapter 8. Riemann integration.

Partitions and step functions. The definition of integral for step functions. The integral of more general functions. Upper and lower integrals. The integrability of monotonic functions. Riemann sums. The main properties of the integral: linearity and additivity with respect to the interval of integration.

Chapter 9. The relationship between integration and differentiation.

The integrability theorem for continuous functions. The integral as a function of the upper limit. Indefinite integrals. The mean-value theorem. Average value of a function. The derivative of an indefinite integral and the first fundamental theorem of calculus. Primitive functions and the second fundamental theorem of calculus.

Chapter 10. Integration formulas.

Integration by substitution. Integration by parts. Integration by partial fractions.

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