Advanced Calculus

Program

Unit 1. Multiple integrals.

1.1. Double integrals over rectangles, triple integrals over parallelepipeds. Iterated integration: Fubini's theorem. Integration of continuous functions over normal regions in two and three dimensions.

1.2. Change of variables. Polar coordinates; cylindrical and spherical coordinates. Symmetry properties.

1.3. Applications: area, volume, geometric center, center of mass, moment of inertia.

Unit 2. Curves. Line integrals. Green's theorem.

2.1. Curves: implicit and parametric form. Arcs and closed curves. Tangent vector and length of a curve.

2.2. Line integral of a scalar field along a curve. Line integral of a vector field. Independence of the path: conservative fields and gradient vectors.

2.3. Green's theorem in simply connected regions. Sufficient condition for a vector field to be conservative. Potential function. Green's theorem in multiply connected regions.

Unit 3. Three-Dimensional field theory.

3.1. Curl of a vector field. Irrotational and gradient fields. Simply connected regions. Sufficient condition for a vector field to be conservative. Potential function.

3.2. Divergence of a vector field: solenoidal field. Vector potential. Star domains. Sufficient condition for a field to be solenoidal. Vector potential for a solenoidal field.

Unit 4. Surfaces and surface integrals.

4.1. Implicit and parametric representation. Surfaces of revolution and ruled surfaces. Tangent plane and normal vector. Orientable surfaces. Closed surfaces and surfaces with boundary.

4.2. Area of a surface. Surface integral of a scalar field. Flux of a vector field across a surface.

Unit 5. Gauss's and Stokes's theorems.

5.1. Gauss divergence theorem and applications.

5.2. Stokes's theorem and applications.

Bibliography

1. Apostol, T.M. (1980) *Calculus*. Reverté.
2. Burgos, J. de (1995), *Cálculo infinitesimal de varias variables*. McGraw-Hill.
3. Marsden, J.E. and Tromba, A.J. (2004) *Cálculo Vectorial*. Pearson Educación.
4. Dineen, S. (1998) *Multivariate Calculus and Geometry*. Springer.
5. Courant, R. and John, F. (1984) *Introducción al Cálculo y al Análisis Matemático (vol. II)*. Limusa.