



**POLITÉCNICA**



UNIVERSIDAD POLITÉCNICA DE MADRID  
ESCUELA TÉCNICA SUPERIOR DE  
INGENIEROS INDUSTRIALES  
DEPARTAMENTO DE MATEMÁTICA APLICADA  
A LA INGENIERÍA INDUSTRIAL

# Algebra

## Program

### **Unit 1. The vector spaces $\mathbf{R}^n$ and $\mathbf{C}^n$ .**

- 1.1. Definition. Linear combinations. Span. Linear dependence and independence. Subspaces.
- 1.2. Basis. Dimension. Subspaces intersection and sum. Direct sum. Complement subspaces. The Grassmann identity.

### **Unit 2. Matrices and linear systems.**

- 2.1. Matrix operations. Image and null spaces.
- 2.2. Rank. Gaussian operations. Change of basis.
- 2.3. Linear systems. Structure of the solutions. Rouché-Frobenius theorem. Solving systems by Gauss elimination.

### **Unit 3. Inner product and orthogonality.**

- 3.1. The usual inner product and the associated norm in  $\mathbf{R}^n$ . The Cauchy-Schwarz and triangular inequalities.
- 3.2. Orthogonality. The orthogonal complement. Orthogonal Projection theorem. Orthogonal sets. Orthonormal basis. Orthogonal matrices. Gram-Schmidt process. QR factorization.
- 3.3. Extension to  $\mathbf{C}^n$ .

### **Unit 4. Orthogonal projections and applications.**

- 4.1. Orthogonal projection onto a subspace.
- 4.2. The least squares problem. Least squares solutions. Least squares and minimum norm.



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4.3. Orthogonal symmetry with respect to a subspace.

4.4. The cross product in  $\mathbf{R}^3$ .

4.5. Rotations in  $\mathbf{R}^2$  and  $\mathbf{R}^3$ .

### **Unit 5. Similarity transformations.**

5.1. Introduction.

5.2. Similarity and diagonalizable matrices.

5.3. Eigenvalues and eigenvectors. The characteristic polynomial.

5.4. Diagonalization. Cayley-Hamilton theorem. Applications.

### **Unit 6. Normal matrices.**

6.1. Unitary similarity and unitary diagonalization.

6.2. Normal matrices.

6.3. Spectral theorem: Hermitian, skew-Hermitian and unitary matrices.

6.4. Real and symmetric matrices. Rayleigh quotient.

### **Unit 7. Singular value decomposition (SVD).**

7.1. The SVD of a matrix. Existence and properties of the factorization. Largest and smallest singular values of a matrix. Moore-Penrose generalized inverse.

7.2. Condition number.

7.3. Norms for vectors and matrices. Matrix norm induced by a vector norm.

### **Bibliography**

1. Burgos, J. de (2006), *Álgebra Lineal y geometría cartesiana*. McGraw-Hill.



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2. Burgos, J. de (2007), *Álgebra Lineal: definiciones, teoremas y resultados*, García Maroto.
3. Noble B. and Daniel J. W. (1988), *Applied Linear Algebra*, Prentice-Hall.