

# Program of Métodos Matemáticos de la Especialidad (Técnicas Energéticas)

## Chapter 1 Lagrange polynomial interpolation in 1D

- 1.1) Introduction.
- 1.2) Definition of Lagrange interpolation polynomial.
- 1.3) Matrix method to calculate the interpolation polynomial.
- 1.4) Lagrange interpolation formula.
- 1.5) Lagrange bases.
- 1.6) Newton formula of Lagrange polynomial.
- 1.7) Divided differences.
- 1.8) Some properties of divided differences.
- 1.9) Error of polynomial interpolation.
- 1.10) Pros and cons of polynomial interpolation.
- 1.11) Chebyshev points as good points for polynomial interpolation.

## Chapter 2 Hermite interpolation

- 2.1) Introduction.
- 2.2) Definition of Hermite interpolation polynomial.
- 2.3) Matrix method to calculate the Hermite interpolation polynomial.
- 2.4) Hermite polynomial in terms of Lagrange bases.
- 2.4) Hermite polynomial and divided differences
- 2.5) The interpolation error.

## Chapter 3 Finite element interpolation in 1D

- 3.1) Introduction: Why piecewise interpolation?
- 3.2) Partition of the finite interval  $I \subset \mathbb{R}$ : nodes, elements and features of a good mesh.
- 3.3) Storage of mesh data.
- 3.4) Lagrange and Hermite formulas of the piecewise interpolation polynomial in a generic element.
- 3.5) Finite element spaces.
- 3.6) Local and global bases.
- 3.7) Global representation of the piecewise interpolation polynomial.
- 3.8) Element of reference and bijective transformations.
- 3.9) The finite element interpolation error.

## Chapter 4 Lagrange finite element interpolation in 2D

- 4.1) Introduction.
- 4.2) Admissible meshes
- 4.3) Storage of mesh data.
- 4.4) Lagrange interpolation in a triangle
- 4.5) Lagrange interpolation in a rectangle.
- 4.6) The reference element.

- 4.7) Affine and isoparametric transformations.
- 4.8) Finite element spaces.
- 4.9) Local and global bases.
- 4.10) The interpolation error.

### **Chapter 5 Numerical calculation of definite integrals**

- 5.1) Introduction.
- 5.2) Definition and properties of a quadrature rule.
- 5.3) Some simple but useful quadrature rules in 1D.
- 5.4) Generation of quadrature rules in 1D: Newton-Cotes rules.
- 5.5) Gaussian quadrature in 1D.
- 5.7) The error of quadrature rules in 1D.
- 5.6) Composed quadrature.
- 5.8) Some quadrature rules for triangles and squares.

### **Chapter 6 Functional setting for linear partial differential equations**

- 6.1) Multidimensional integration by parts.
- 6.2) Weak derivatives.
- 6.3) Introduction to functional spaces  $L^2$ ,  $H^m$ ,  $m \geq 1$ .

### **Chapter 7 Second order linear elliptic problems**

- 7.1) Definition of second order linear elliptic problems.
- 7.2) Engineering examples.
- 7.3) Definition of the classical solution: Examples.
- 7.4) Definition of the weak solution: Why the weak solution is the right solution in engineering problems?
- \*7.5) Bilinear form and linear functional associated with the weak solution.
- \*7.6) Existence and uniqueness of the weak solution.

### **Chapter 8 Finite element solution of second order elliptic problems**

- 8.1) Construction of finite element spaces.
- 8.2) Finite element formulation.
- 8.3) Matrix formulation of the finite element solution.
- 8.4) Case study 1: step by step of the numerical solution of a one-dimensional elliptic problem.
- \*8.5) Case study 2: step by step of the numerical solution of a two-dimensional elliptic problem.
- \*8.6) Error analysis.

### **Chapter 9 Evolution problems I: linear parabolic problems**

- 9.1) The parabolic model problem.
- 9.2) Definition of the classical solution: examples.
- 9.3) Definition of the weak solution and study of its more important properties.

9.4) Time discretization by the implicit and explicit Euler schemes and Crank-Nicolson scheme.

9.5) Finite element formulation of the time discretized problem.

9.6) Stability of the numerical solution.

9.7) Case study 1: step by step of the numerical solution of a one-dimensional parabolic problem.

\*9.8) Case study 2: step by step of the numerical solution of a two-dimensional parabolic problem.

### **Chapter 10 Evolution problems II: second order hyperbolic problems**

10.1) The second order wave equation.

10.2) Time discretization of the second order wave equation: explicit schemes and Newmark scheme

10.3) Finite element solution of the time discretized equation.

10.4) Stability of the numerical solution.

10.5) Case study 1: step by step of the numerical solution of one dimensional wave equation.

### **\*Chapter 11 Elliptic eigenvalue problem**

11.1) Eigenfunction expansions.

11.2) Finite element solution of an elliptic eigenvalue problem.

### **\*Chapter 12 Introduction to the numerical solution of inverse problems**

12.1) Introduction to inverse problems.

12.2) A simple one-dimensional linear problem.

12.3) Inverse problems with time independent parameters.